SN – 349

I Semester B.A./B.Sc. Examination, November/December 2017
(CBCS) (F+R) (2014 –15 & Onwards)
MATHEMATICS – I

Time : 3 Hours Max. Marks : 70

Instruction : *Answer **all** questions.*

PART – A

Answer **any five** questions. **(5×2 = 10)**

1. a) Transform the matrix $\begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix}$ into $\begin{bmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & 0 & 0 \end{bmatrix}$ using elementary transformations.

   b) Find the eigen values of the matrix $\begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}$.

   c) Find the n^{th} derivative of $\log_e(3x-2)$.

   d) If $z = e^{\frac{x}{y}}$, find $\frac{\partial^2 z}{\partial x \partial y}$.

   e) Evaluate $\int_0^{\pi} \sin^5 x \, dx$.

   f) Evaluate $\int_0^{\infty} \frac{x^2}{(1+x^2)^2} dx$.

   g) Show that the plane $2x - 2y + z + 12 = 0$ touches the sphere $x^2 + y^2 + z^2 - 2x - 4y - 2z - 3 = 0$.

   h) Find 'k' so that the spheres $x^2 + y^2 + z^2 + 4x + ky + 2z + 6 = 0$ and $x^2 + y^2 + z^2 + 2x - 4y - 2z + 6 = 0$ may be orthogonal.

P.T.O.

PART – B

Answer **one full** question. **(1×15=15)**

2. a) Find rank of the matrix by reducing in to echelon form.

$$\begin{bmatrix} 1 & 0 & 2 & -2 \\ 2 & -1 & 0 & -1 \\ 1 & 0 & 2 & -1 \\ 4 & -1 & 3 & -1 \end{bmatrix}$$

b) Solve completely the system of equations $x + 3y - 2z = 0$; $2x - y + 4z = 0$; $x - 11y + 14z = 0$.

c) Find the eigen values and eigen vectors of the matrix $\begin{bmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{bmatrix}$.

OR

3. a) Find the values of λ and μ such that the equations $x + y + z = 6$; $x + 2y + 3z = 10$; $x + 2y + \lambda = \mu$ have i) a unique solution ii) infinite number of solutions.

b) Reduce the matrix $\begin{bmatrix} 4 & 0 & 2 & 1 \\ 2 & 1 & 3 & 4 \\ 2 & 3 & 4 & 7 \end{bmatrix}$ into normal form and find its rank.

c) For the matrix $A = \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & -1 \\ 3 & -1 & 1 \end{bmatrix}$, find A^{-1} using Cayley-Hamilton theorem.

PART – C

Answer **two full** questions. **(2×15 = 30)**

4. a) Find the n^{th} derivative of $\frac{x^2}{(x-1)^2(x-2)}$.

   b) Find the n^{th} derivatives of
   i) $(x^2 + 1)\, e^{5x}$ ii) $\cos 3x \sin 4x$.

   c) If $y = (\sin^{-1}x)^2$, prove that $(1 - x^2)\, y_{n+2} - (2n + 1)\, xy_{n+1} - n^2 y_n = 0$.

OR

5. a) If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = \frac{3}{x+y+z}$.

   b) State and prove Euler's theorem for homogeneous function of x and y.

   c) Find $\frac{dz}{dt}$, if $z = \log(x^2 - y^2)$, where $x = e^t \cos t$, $y = e^t \sin t$.

6. a) If $u = f(y - z, z - x, x - y)$, show that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$.

   b) If $u = \frac{yz}{x}, v = \frac{xz}{y}, w = \frac{xy}{z}$ show that $\frac{\partial(u,v,w)}{\partial(x,y,z)} = 4$.

   c) For any positive integer n, prove that

$$\int_0^{\frac{\pi}{2}} \cos^n x\,dx = \frac{(n-1)(n-3)(n-5)\ldots 2 \text{ or } 1}{n(n-2)(n-4)\ldots 2 \text{ or } 1} \cdot R$$

   Where $R = \frac{\pi}{2}$, if n is even

   = 1, if n is odd.

OR

7. a) Obtain the reduction for $\int \sec^n x dx$

b) Evaluate $\int_0^{2a} x^2\sqrt{2ax-x^2}\, dx$.

c) Evaluate $\int_0^{\infty} \frac{e^{-x}\sin\alpha}{x} dx$, where α is a parameter using Leibnitz's rule of differentiation under integral sign.

PART – D

Answer **one full** question. **(1×15 = 15)**

8. a) Find the equation of the plane passing through (2, 2, 1) and (9, 3, 6) and perpendicular to the plane $2x + 6y + 6z = 9$.

b) Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-4}{5} = \frac{y-5}{6} = \frac{z-6}{7}$ are coplanar and find the point of intersection.

c) Find the equation of sphere passing through the points (3, 0, 0), (0, –1, 0), (0, 0, –2) and having its centre on the plane $3x + 2y + 4z - 1 = 0$.

OR

9. a) Show that the lines $\frac{x-1}{2} = \frac{y-2}{2} = \frac{z-3}{1}$ and $\frac{x-2}{3} = \frac{y-2}{2} = \frac{z-6}{4}$ are coplanar. Find the equation of the plane containing these lines.

b) Derive the equation of a right circular cone in the standard form $x^2 + y^2 = z^2 \tan^2\alpha$.

c) Find the equation of right circular cylinder whose radius is 4 units and passes through (1, –2, 3) and (3, –1, 1).
