Q.P. Code : 11123

First Semester B.Sc. Degree Examination, November/December 2019

(Semester Scheme – CBCS – Freshers and Repeaters – 2018-19 Onwards)

MATHEMATICS – I

Time : 3 Hours] *[Max. Marks : 70*

Instructions to Candidates : Answer all questions.

PART – A

Answer any **FIVE** questions. **(5 × 2 = 10)**

1. (a) Find the value of 'a' in order that the matrix $A = \begin{pmatrix} 6 & a & -1 \\ 5 & 3 & 1 \\ 4 & 3 & 2 \end{pmatrix}$ has rank 2.

(b) Verify whether the system of equations $x + y - 2z = 5$, $x - 2y + z = -2$, $-2x + y + z = 4$ are consistent.

(c) Find the n^{th} order derivative of $y = e^{mx+c}$

(d) If $u = x\sin y + y\cos x$ find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$.

(e) Evaluate $\int_0^{\pi/2} \sin^4 x \, dx$.

(f) Evaluate $\int_0^{\pi/2} \sin^4\theta \cos^3\theta \, d\theta$.

(g) Find the angle between the planes $\vec{r}.(2\hat{i} + 2\hat{j} - 3\hat{k}) = 5$ and $\vec{r}.(3\hat{i} - 3\hat{j} + 5\hat{k}) = 6$.

(h) If two spheres $x^2 + y^2 + z^2 + 6z - k = 0$ and $x^2 + y^2 + z^2 + 10y - 4z - 8 = 0$ cuts orthogonally, find k.

PART – B

Answer **ONE** full questions : **(1 × 15 = 15)**

2. (a) Reduce the matrix $\begin{pmatrix} 1 & 1 & 1 & 6 \\ 1 & -1 & 2 & 5 \\ 3 & 1 & 1 & 8 \end{pmatrix}$ to its normal form and find its rank.

(b) Solve the system of equations $x + y = 0,\ x - y - z = 0,\ 3x + y - z = 0$.

(c) Find the eigen value and eigen vectors of the matrix $\begin{pmatrix} 4 & 1 \\ -1 & 2 \end{pmatrix}$.

Or

3. (a) Find the rank of the matrix :

$$A = \begin{pmatrix} 1 & 3 & -1 & 2 \\ 0 & 11 & -5 & 3 \\ 2 & -5 & 3 & 1 \\ 4 & 1 & 1 & 5 \end{pmatrix}$$

by row reduced echelon form.

(b) Test the following system of equations $x + 2y - z = 3,\ 3x - y + 2z = 1,\ 2x - 2y + 3z = 2$ for consistency and if consistent solve.

(c) Verify Cayley-Hamilton theorem for the matrix $A = \begin{pmatrix} 4 & -1 \\ 1 & 2 \end{pmatrix}$.

PART – C

Answer **TWO** full questions : **(2 × 15 = 30)**

4. (a) Find nth derivative of $\dfrac{1}{x^2 - 6x + 8}$.

(b) Find the n^{th} derivative of $e^{3x}\sin^2 x$.

(c) If $y = (\sin^{-1} x)^2$ show that $(1-x^2)y_{n+2} - (2n+1)y_{n+1} - n^2 y_n = 0$.

xOr

5. (a) If $u = (x-y)^n + (y-z)^n + (z-x)^n$ prove that $\dfrac{\partial u}{\partial x} + \dfrac{\partial u}{\partial y} + \dfrac{\partial u}{\partial z} = 0$.

(b) If $f = x^y + y^x$ find $\dfrac{\partial^2 f}{\partial x \partial y}$ and $\dfrac{\partial^2 f}{\partial x^2}$.

(c) If $u = \tan^{-1}\left(\dfrac{x^2+y^2}{x+y}\right)$ show that $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y} = \dfrac{1}{2}\sin 2u$.

6. (a) If $u(x, y) = x\cos y + y\sin x$, where $x = \sin 2t$, $y = \cos 2t$ find $\frac{du}{dt}$.

(b) If $u = xy, v = yz, w = zx$ prove that $\frac{\partial(u, v, w)}{\partial(x, y, z)} = 2\,xyz$.

(c) Obtain the reduction formula for $\int \sin^n x\,dx$, where n is a positive integer.

Or

7. (a) Obtain the reduction formula for $\int \cot^n x\,dx$ where n is a positive integer.

(b) Evaluate $\int_0^{2a} x^2\sqrt{2ax - x^2}\,dx$.

(c) Evaluate by using Leibnitz's rule of differentiation under the integral sign for $\int_0^{\pi/2} \frac{1}{\alpha(1 + \cos x)}\,dx$, where α is a parameter.

PART – D

Answer **ONE** full question : **(1 × 15 = 15)**

8. (a) Find the equation of the plane passing through the line of intersection of the planes $x + 2y + 3z - 4 = 0$, $2x + y - z + 5 = 0$ and perpendicular to the plane $5x + 3y + 6z + 8 = 0$.

(b) Find the shortest distance between the skew lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-6}{4} = \frac{z-5}{5}$.

(c) Find the equation of the right circular cylinder of radius 2 and whose axis is $\frac{x-1}{2} = \frac{y+3}{-1} = \frac{z-3}{5}$.

Or

9. (a) Show that the lines $\frac{x-1}{1} = \frac{y+1}{-1} = \frac{z-3}{1}$ and $\frac{x-2}{2} = \frac{y-4}{1} = \frac{z-6}{3}$ are coplanar. Find also the equation of the plane containing them.

(b) Find the equation of the sphere which passes through the points $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$ and whose centre lies on the plane $3x - y - z = 2$.

(c) Find the equation of the right circular cone generated by revolving the line $\frac{x-1}{-1} = \frac{y-2}{3} = \frac{z-3}{3}$ about the line $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$.
