

Q.P. Code : 11323

51819101

**Third Semester B.Sc. Degree Examination,
November/December 2019**

(Semester Scheme – CBCS – Freshers – 2019-20)

MATHEMATICS – III

Time : 3 Hours]

[Max. Marks : 70

Instructions to Candidates : Answer all questions.

PART – A

Answer any **FIVE** questions :

(5 × 2 = 10)

1. (a) Define order of an element of a group.
(b) Find the number of generators of a cyclic group of order 24.
(c) Test the nature of the sequence $\{1 + \cos n\pi\}$. *oscill*
(d) State Raabe's Test for series of Positive terms.
(e) Test the convergence of the series

$$1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \dots$$

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- (f) State Rolle's theorem.
(g) Find Left hand Limit of

$$f(x) = \begin{cases} 1+x & \text{for } x \leq 2 \\ 5-x & \text{for } x > 2 \end{cases} \text{ at } x = 2.$$

- (h) Evaluate $\lim_{x \rightarrow 0} \left(\frac{a^x - b^x}{x} \right)$.

PART – B

Answer any **ONE** full question :

(1 × 15 = 15)

2. (a) Prove that if 'a' is any element of a group G of order n then $a^m = e$ for any integer m if and only if n divides m.
(b) Prove that subgroup of a cyclic group is cyclic.
(c) Find all the right cosets of the subgroup $H = \{0, 4, 8\}$ in Z_{12} and also verify Lagrange's theorem.

Or

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3. (a) If a and x are any two elements of a group G then prove that $O(a) = O(xax^{-1})$.
- (b) Prove that the group $G = \{1, 2, 3, 4, 5, 6\}$ under multiplication modulo 7 is cyclic and find its generators.
- (c) If G is any finite group and H is any subgroup of G then prove that $O(H)$ divides $O(G)$.

PART - C

Answer any **TWO** full questions :

(2 × 15 = 30)

4. (a) Prove that a monotonically increasing sequence which is bounded above is convergent.
- (b) Discuss the nature of the sequence $\left\{ \left(1 + \frac{1}{n}\right)^n \right\}$.
- (c) Examine the convergence of the sequences :
- (i) $\left\{ \sqrt{n+1} - \sqrt{n} \right\}$
- (ii) $\left\{ \frac{(n+1)^{n+1}}{n^n} \right\}$.

Or

5. (a) Prove that every convergent sequence is bounded.
- (b) If $\lim_{n \rightarrow \infty} a_n = a$ and $\lim_{n \rightarrow \infty} b_n = b$ then prove that $\lim_{n \rightarrow \infty} (a_n + b_n) = (a + b)$.
- (c) Discuss the nature of the sequence $\{a_n\}$ where $a_n = \frac{1}{n+1} + \frac{1}{n+2} + \dots + \frac{1}{n+n}$.
6. (a) State and prove D'Alembert's Ratio Test for series of positive terms.
- (b) Test the Convergence of the series $1 + \frac{1}{2} + \frac{1.3}{2.4} + \frac{1.3.5}{2.4.6} + \dots \infty$.
- (c) Sum the series to infinity $1 + \frac{2}{6} + \frac{2.5}{6.12} + \frac{2.5.8}{6.12.18} + \dots$

Or

7. (a) State and prove Cauchy's Root Test for series of positive terms.
- (b) Discuss the convergence of the series $\sum_{n=1}^{\infty} \left(\frac{nx}{n+1} \right)^n$.
- (c) Sum the series to infinity $\frac{1^2}{1!} + \frac{2^2}{2!} + \frac{3^2}{3!} + \dots$

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PART - D

Answer **ONE** full question :

(1 × 15 = 15)

8. (a) Prove that a function which is continuous in a closed interval attains its bounds.
- (b) Verify Rolle's theorem for the function $f(x) = x(x-3)^2$ in $[0, 3]$.
- (c) Evaluate $\lim_{x \rightarrow \frac{\pi}{2}} (\sec x)^{\cot x}$.

Or

9. (a) Examine the differentiability of the function $f(x) = \begin{cases} x^2 & \text{if } x \leq 3 \\ 6x-9 & \text{if } x > 3 \end{cases}$ at $x = 3$.
- (b) State and prove Lagrange's mean value theorem.
- (c) Using Maclaurin's expansion prove that

$$\log(1+x) = 1 - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \dots$$
