Q.P. Code : 11223

Second Semester B.Sc. Degree Examination, May/June 2019

(CBCS Scheme)

Mathematics

Paper II — MATHEMATICS

Time : 3 Hours] *[Max. Marks : 70*

*Instructions to Candidates : Answers **ALL** questions.*

PART – A

1. Answer any **FIVE** questions : **(5 × 2 = 10)**

   (a) Let $*$ be an binary operation on the set of real numbers R defined by $a * b = a + b - 7$, $\forall\ a, b \in R$. Is $*$ is commutative?

   (b) Prove that in a group $(G, *)$, $(a^{-1})^{-1} = a$, $\forall\ a \in G$.

   (c) Find $\frac{dS}{dx}$ for the curve $y^2 = 4ax$.

   (d) Find the polar subnormal for the curve $r\theta = a$.

   (e) Find the asymptotes parallel to the coordinate axes for the curve $xy^3 - x^2y = x^2 + 1$.

   (f) Find the arc length of the curve $y = c\ \cosh\left(\frac{x}{c}\right)$ from $x = 0$ to $x = a$.

   (g) Show that the equation $(x^2 - 2xy + 3y^2)\,dx + (y^2 + 6xy - x^2)\,dy = 0$ is exact.

   (h) Solve : $p^2 - 5p + 6 = 0$, where $p = \frac{dy}{dx}$.

PART – B

Answer **ONE** full question : **(1 × 15 = 15)**

2. (a) If $(G, *)$ is a group, then prove that $(a * b)^{-1} = b^{-1} * a^{-1}$, $\forall\ a, b \in G$.

   (b) Show that the set $G = \{0, 1, 2, 3, 4, 5, 6\}$ is an abelian group with respect to addition modulo 7.

   (c) For the set $A = \{1, 2, 3\}$, where $f = \begin{pmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{pmatrix}$ and $g = \begin{pmatrix} 1 & 2 & 3 \\ 3 & 1 & 2 \end{pmatrix}$ then show that $(f \circ g)$ is the identity element and find $(f^{-1} \circ g^{-1})$.

Or

3. (a) If in a group G, $(ab)^2 = a^2 b^2$, $\forall\ a, b \in G$. Prove that G is abelian.

   (b) Show that the set C of all complex numbers is a group under addition of complex numbers.

   (c) Show that $H = \{1, 2, 4\}$ is a subgroup of the group $G = \{1, 2, 3, 4, 5, 6\}$ under multiplication modulo 7.

PART – C

Answer any **TWO** full questions : **(2 × 15 = 30)**

4. (a) With usual notation prove that $\tan\phi = r\dfrac{d\theta}{dr}$ for the polar curve $r = f(\theta)$.

   (b) Find the radius of curvature at a point t on the curve $x = a\cos^3 t$, $y = a\sin^3 t$.

   (c) Find the Pedal equation of the curve $r^2 = a^2 \cos 2\theta$.

Or

5. (a) Find the envelope of the family of straight lines $y = \alpha\, x + \dfrac{a}{\alpha}$ where α is a parameter.

   (b) Show that the curves $r = a(1 + \sin\theta)$ and $r = b(1 - \sin\theta)$ intersect orthogonally.

   (c) Prove that with usual notations, the radius of curvature of the curve $x = f(t)$, $y = g(t)$ is

$$\rho = \frac{[(\dot{x})^2 + (\dot{y})^2]^{\frac{3}{2}}}{[\dot{x}\,\ddot{y} - \dot{y}\,\ddot{x}]}$$

6. (a) Find all the asymptotes of the curve
$$2x^3 - x^2y - 2xy^2 + y^3 - 4x^2 + 8xy - 4x + 1 = 0 .$$

(b) Find the evolute of the parabola $y^2 = 4ax$.

(c) Find the position and nature of the double points of the curve $y^2 = x^2(x-1)$.

Or

7. (a) Find the perimeter of the cardioids $r = a(1+\cos\theta)$.

(b) Find the surface area of the solid generated by revolution of the curve $x = a\cos^3 t$, $y = a\sin^3 t$ about the x-axis.

(c) Find the volume generated by revolving the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ about x-axis.

PART – D

Answer **ONE** full question : **(1 × 15 = 15)**

8. (a) Solve : $\frac{dy}{dx} - y \sec x\ x = y^3 \tan x$

(b) Solve : $p^2 + 2py\cot x - y^2 = 0$

(c) Find the orthogonal trajectories of the family of parabolas $y = ax^2$, where a is a parameter.

Or

9. (a) Solve : $\frac{dy}{dx} + \frac{2x}{1+x^2} y = \frac{1}{(1+x^2)^2}$

(b) Verify for exactness and solve $(ax + hy + g)\, dx + (hx + by + f)\, dy = 0$.

(c) Find the general and singular solution of $\sin px \cos y = \cos px \sin y + p$.