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# GS-324

## VI Semester B.A./B.Sc. Examination, May/June - 2019

### MATHEMATICS

### Mathematics - VIII

### (CBCS) (Fresh+Repeaters) (2016-17 & Onwards)

Time : 3 Hours Max. Marks : 70

**Instruction :** Answer **all** questions.

**PART - A**

**1.** Answer **any five** questions. **5x2=10**

(a) Evaluate $\lim_{z\to 1+i}(z^2+2z)$.

(b) Show that $\left|\frac{z-2}{z+2}\right|=3$ represents a circle.

(c) Show that $u=e^x \sin y+x^2-y^2$ is harmonic.

(d) Define cross ratio of four points.

(e) State Liouville's Theorem.

(f) Evaluate $\phi(\bar{z})^2\,dz$ around the circle $|z|=1$

(g) Write Euler modified formula.

(h) State Runge- Kutta Method of order 4.

**PART- B**

Answer **four** full Questions. **4x10=40**

**2.** (a) Show that $\arg\left(\frac{z-1+i}{z+1}\right)=\frac{\pi}{4}$

(b) Prove that $\frac{\partial u}{\partial r}=\frac{1}{r}\frac{\partial v}{\partial \theta}$ and $\frac{\partial u}{\partial \theta}=-\frac{\partial v}{\partial r}$

**OR**

**3.** (a) Prove that $\lim_{z\to 0}\left(\frac{\bar{z}}{z}\right)$ does not exists.

(b) Show that $f(z)=\sin z$ is analytic and hence prove that $f'(z)=\cos z$.

**4.** (a) Show that $u=y^3-3x^2y$ is harmonic and find its harmonic conjugate.

(b) If $f(z)=u+iv$ is an analytic function then prove that the curves $u(x,y)=c_1$, $v(x,y)=c_2$ form two orthogonal families.

**OR**

**P.T.O.**

5. (a) Find the analytic function $f(z)=u+iv$ given that $u-v=e^x(\cos y-\sin y)$.

(b) If $f(z)=u+iv$ is analytic then show that $\left[\frac{\partial}{\partial x}|f(z)|\right]^2+\left[\frac{\partial}{\partial y}|f(z)|\right]^2=|f'(z)|^2$

6. (a) Evaluate $\int_{(0,3)}^{(2,4)}[(2y+x^2)dx+(3x-y)dy]$ using the substitution $x=2t,\ y=t^2+3$.

(b) State and Prove Cauchy's Integral Formula.

**OR**

7. (a) Evaluate $\oint_c \frac{z-4}{z(z^2+9)}dz$ where c is the circle $|z|=1$.

(b) State and prove Cauchy's Integral Theorem.

8. (a) Prove that the Bilinear Transformation preserves the cross ratio of four points.

(b) Discuss the transformation $w=z^2$.

**OR**

9. (a) Find the Bilinear Transformation which maps $z=1,e^1,-1$ on to $w=i,0,-i$.

(b) Show that the transformation $W=1/z$ transforms a circle to a circle or to a straight line.

**PART- C**

Answer **two** full questions. **2x10=20**

10. (a) Find the root of the equation $x^3-4x+1=0$ by Regula-falsi method upto three decimal places.

(b) Use Newton-Raphson Method to find a real root of the equation. $x^3-9x+1=0$ near $x=3$.

**OR**

11. (a) Solve by Guass- Jacobi method.

$10x+2y+z=9$

$x+10y-z=-22$

$-2x+3y+10z=22$

(b) Find the largest Eigen Value of the matrix.

$$A=\begin{pmatrix}1 & 6 & 1\\ 1 & 2 & 0\\ 0 & 0 & 2\end{pmatrix}$$

**12.** (a) Use Taylor's series method to find $y$ at $x=0.1$ considering terms upto the third degree given $\frac{dy}{dx} = x^2+y^2$ and $y(0)=1$.

(b) Using Euler's modified method, find $y(0.2)$ given $\frac{dy}{dx} = x-y^2$, $y(0)=1$ taking $h=0.1$.

**OR**

**13.** (a) Use Euler's Method to solve $\frac{dy}{dx} = x+y$, $y(0)=1$ for $x=0.0\ (0.2)1.0$

(b) Using Runge-Kutta Method find $y(0.2)$ for $\frac{dy}{dx} = x+y$; $y(0)=1$ taking $h=0.2$.

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